ALGEBRA

Workbook

Foreword

One of the prime objectives of education is to develop thinking skill in learners. Thinking skills is essential to success in education, career and life in general. Mathematical reasoning is one of the essential skills to be an effective critical thinker. An individual who is adept at mathematical reasoning is able

- to read between the numbers to deduce patterns
- to relate various parameters and arrive at a relationship specific to the problem
- to construct alternate scenarios with the same parameters thereby resulting in multiple solutions
- to correlate a known or an analogous formula/theorem to the given problem
- to converge at a solution using different approaches
- to convert a problem from the given form into another less complex form

We are convinced that the preparation towards the complex area of mathematical reasoning should comprise more than referring to discussion of sample questions. Therefore this module attempts to present before you learning experiences which will empower you with essential mathematical reasoning and problem solving tools.

This module is presented in a format which aims to create a virtual teacher who

would hand hold you while you explore the realm of math. The concepts are explained in a lucid manner with minimalist words and maximum transfer learning.

It takes more than one reading to fully assimilate and appreciate the concepts.

While solving problems, initially use the pen and paper extensively. After you learn computational techniques try to 'visual think' the computation.

Remember math learning is not about remembering formula or tables or theorems, it is all about learning the processes involved in successful problem solving.
Enjoy learning

1. Simple linear equations and simultaneous equations

A polynomial equation is of the form

$ax^n+bx^{n-1}+cx^{n-2}+.....f$ where a,b,c,d....f are constants.

Other types of equations are

Monomial(only one term in the equation) : ax, bx^2, cx^3

Binomial(only two terms in the equation) : $ax+ bx^2$, $cx^3 + dx$, $gx+ 4$

Trinomial(only three terms in the equation) : $ax+ bx^2+c$, $cx^3 + dx^2 +rx$,

1.1 Coefficient of x

In a polynomial equation $ax^n+bx^{n-1}+cx^{n-2}+.....f$

a,b,c,d....f are the coefficients of $x^n, x^{n-1}, x^{n-2}...x^o$

Practice Exercise 1.1

Write the coefficients of

1. x^4 in $3x^2 + 4x+ 5$ _____

2. x^2 in $5x^2 + 4x+ 6$ _____

3. x^3 in $3x^4 +4x^3 + 5x^2+ 5$ _____

4. x^o in $3x^4 +4x^3 + 5x^2+ 5$ _____

5. x in $3x^4 +4x^3 + 5x^2+ 5x + 4$ _____

1.2 Simultaneous equations

Simultaneous equations are pairs of equations with unknowns.

For example 2x +4y = 9 and

3x + 6y =20

Steps to solve simultaneous equations

Step 1

Write the equations such that similar terms are place one below the other

2x + 4y = 9 ---- (1)

3x + 7y = 20---(2)

Here 2x and 3x are placed one below the other

Step 2

Choose one of unknown terms(x or y). Multiply each equation with a term such that the coefficient of the chosen unknown term is the same.

2x + 4y = 9 (x) 3

3x + 7y = 20 (x) 2

In this example, x is chosen.

Multiply equation 1 with 3 and equation 2 with 2 such that in both cases the coefficient of x is 6.

6x + 12y = 27

6x + 14y = 40

Step 3

Eliminate the chosen term either by subtracting or adding the equations. One unknown term can be determined.

6x + 12y = 27

(-) 6x + 14y = 40

-2y = -13

Here both the equations are subtracted to eliminate x. $y = 13/2$

Step 4

Substitute the value of known parameter in Any of the equation to determine the other value.

$6x + 12y = 27$

$6x + 12(13/2) = 27$

$x = -51/6$

Practice Exercise 1.2

Solve for the unknown variable

1. $3x + 23 = 56$ _____
2. $8x + \frac{21}{4} = 56$ _____
3. $2x - (3x-4) = 56$ _____
4. $\frac{x}{3x-2} = 5$ _____
5. $\frac{4}{x-2} = \frac{56}{2x-3}$ _____
6. $\frac{3x-4}{x-2} = \frac{56}{2}$ _____
7. $15(x-3) + 23(x-2) = 34x-30$ _____
8. $\frac{2}{x-2} = \frac{4}{x+2}$ _____
9. $\frac{x-2}{3} + \frac{5x-34}{5} = \frac{1}{32}$ _____
10. $5x + 3 = \frac{27}{4}(x-2)$ _____

1.3 Equation based word problems

Solve for the unknown variable.

For example:

A number added to its two-thirds is 20. Find the number.

Let x be the number

The equation can be written as

$x + \frac{2}{3}x = 20$. Hence x = 12

Practice Exercise 1.3

Use linear/simultaneous equations to arrive at an answer

1. One fifth of a number decreased by 4 is equal to 6. Find the number.

   _____

2. Divide 50 into two parts such that one part is 4 times the other.

   _____

3. The sum of two numbers is 50, and their difference is 20. Find the numbers.

   _____

4. A is 20 years older than B. He is also 6 times as old as B. Find their ages.

   _____

5. A number is 2/5 times the other. If their sum is 70, find the two number.

   _____

6. Find n if n is sum of consecutive odd positive integers whose sum is 81.

   _____

7. Find four consecutive multiples of 7 whose sum is 294.

   _____

8. The ratio between two numbers is 11 : 13. If the smaller number is 286, find the number. _____

9. A number whose fifth part increased by 5 is equal to its fourth part diminished by 5. Find the number.

_____

10. Four-fifth of a number is more than three fourth of the number by 4. Find the number.

_____

11. Find three consecutive numbers such that twice the first, 3 times the second and 4 times the third together make 182.

_____

12. The difference between two numbers is 1365. When the larger number is divided by the smaller one, the quotient is 6 and the remainder is 15. Find the numbers.

_____

13. The breadth of a rectangular room is 2 m less than its length. If the perimeter of the room is 16 m, find the lengh and breadth of the room.

_____

14. The length of a rectangular field is twice its breadth. If the perimeter of the field be 1500 m, find its dimensions.

_____

15. The length of a room exceeds its breadth by 3 m. If both, the length and the breadth, are increased by 1 metre, then the area of the room is increased by 18 m^2.

Find the length and breadth of the room.

_____

2. Quadratic equations

Equations of the form $ax^2 + bx + c = 0$ are called quadratic equations.

2.1 Solving quadratic equations – using factors

Quadratic equations can be solved by determining two numbers(p,q) such that $p + q = -\frac{b}{a}$ and $p.q = \frac{c}{a}$

where a,b,c are the coefficients of $ax^2 + bx + c$

For example: To solve $x^2 + 5x + 6 = 0$

Determine two numbers which add to 5 and whose product is 6

The numbers are 2,3

$x^2 + 5x + 6 = x^2 + 3x + 2x + 6 = 0$

$= x(x + 3) + 2(x + 3)$

$= (x+3)(x+2) = 0$

The roots are

$x + 3 = 0$ and $x + 2 = 0$

$x = -3$ and $x = -2$

Practice Exercise 2.1

Solve

1. $x^2 + 5x + 4$ _____

2. $x^2 + 8x + 12$ _____

3. $x^2 - 9x + 36$ _____

4. $x^2 - 15x + 54$ _____

5. $x^2 + 15x + 126$ _____

6. $x^2 + 7x - 18$ _____

7. $x^2 + 3x - 18$ _____

8. $x^2 - 3x - 10$ _____

9. $x^2 - 5x - 24$ _____

10. $x^2 - 2x - 120$ _____

2.2 Solving quadratic equations – using formula

Quadratic equations can be solved by using the formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where a,b,c are the coefficients of $ax^2 + bx + c = 0$

For example: To solve $x^2 + 5x + 6 = 0$

$a = 1, b = 5, c = 6$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-5 \pm \sqrt{5^2 - 4.1.6}}{2.6}$

$x = \frac{-5 \pm \sqrt{1}}{12} = \frac{-5+1}{12}, \frac{-5-1}{12}$

$x = \frac{-4}{12}, \frac{-6}{12}$, -1/3 and -1/2

Practice Exercise 2.2

Solve using fromula (check with the previous exercise results)

1. $x^2 + 5x + 4$ _____

2. $x^2 + 8x + 12$ _____

3. $x^2 - 9x + 36$ _____

4. $x^2 - 15x + 54$ _____

5. $x^2 + 15x + 126$ _____

6. $x^2 + 7x - 18$ _____

7. $x^2 + 3x - 18$ _____

8. $x^2 - 3x - 10$ _____

9. $x^2 - 5x - 24$ _____

10. $x^2 - 2x - 120$ _____

2.3 Word based problems involving quadratic equations

Quadratic equations can be used to arrive at an answer

Use quadratic equations to solve the following questions

Practice Exercise 2.3

1. John needed a board of area 120 sq.m . He went to a shop to buy it. Shopkeeper gave a board which differed in the dimensions given by John. He asked the shopkeeper to give a board which is 2m more than the board given by shopkeeper in lengthand 3m lesser breadth. Find the shopkeeper's board dimension and dimension of board which John asked for.

_____

2. Maths teacher told John to find sum of 'n' natural number. John missed a number and got the sum as 3932. Find the number he missed?

_____

3. The Sum of number and its square is 306. Find the number.

_____

4. If the number of squares which can be formed from chess board is 385, find the dimensions of chess board

_____

5. Product of a number and three less than twice the number is 665. Find the number.

_____

6. The sum of squares of three consecutive numbers is 194. Find the middle number.

_____

7. In a Right angled triangle of base 4cm, the hypoteneuse is 5cm, find the length of third side

_____

8. Cost of painting square of dimension 9m is Rs. 648. If the side of square is reduced by x m, the cost is lowered by 2.25 times. Find x.

_____

9. The area of two rectangles is 84 sq. m each. The difference between length and breadth of first rectangle is a perfect square. The sum of the length breadth of second Rectangle gives the same perfect square. Find the

dimensions of two rectangle.

_____

10. Ram's age is 5 times his father. The sum of squares of Ram's age and his younger brother age is twice that of the father's age. Find age of Ram's elder brother who is 3 years older than Ram?

_____

11. Ram was very interested in numbers and so he added the integer from 1 up to n. He stopped adding when the sum exceeded 1000. Find n.

_____

12. Ravi and his family went out in bus. Ravi bought the tickets. Price of the ticket was same as number of tickets bought. Ravi gave Rs.100. But the conductor didn't have enough change. So he asked Ravi to wait. Then Ravi decided to buy tickets, for two of his neighbors who had the same problem. The total cost of the ticket was a multiple of 10. Now the conductor gave the change to Ravi. Find the number of people in Ravi's family.

_____

13. Sum of number and is square is 182. Now the number is increased by 5 . The sum of the new number and its square is 342. Find the product of first number and second number.

_____

14. x mangoes costs Rs. 68. x times x mangoes costs Rs. 272. Find cost of a dozen of mangoes.

_____

3. Basic logs and functions

Definition : If $a^x = N$

then the index x is the logarithm of 'N' with respect to base a.

$a^x = N$ can be written as :

$x = \log_a N$

* The logarithm of any number of base 10 is called common logarithm.
* The logarithm of any number to base 'e' is called natural logarithm,

  where e =2.7183

  Observe these examples

  - Convert 81 in terms of log

    Taking log

    log 81 , log $9^2 = 2 \log 9$

  - Convert 2 log9 into the exponential form

    $2 \log 9 = a$

    $2 \log_{10} 9 = a$

    $\log_{10} 9^2 = a$

    Removing log

    $9^2 = 10^a$, $10^a = 81$

Practice Exercise 3.1

1. Represent the numbers in logarithm form:

   (i) 64 _____

   (ii) 27 _____

   (iii) 16 _____

   (iv) $a^y = b$ _____

2. Convert into exponential form:

   (i) $A = \text{Log}_A B$ _____

   (ii) $\log_y x = a$ _____

   (iii) $\log_5 125 = 3$ _____

   (iv) $2 = \log_9 81$ _____

   (v) $3 = \log_4 64$ _____

3.1 Properties of logarithms

* Logarithm of 1 to any base =

  $\text{Log}_b 1 = 0$

* Logarithm of a number to the same number as base = 1

  $\text{Log}_b b = 1$

  Theorems:-

  $\text{Log}(ab) = \log a + \log b$

  $\text{Log } a^b = b \log a$

  $\log\left(\frac{a}{b}\right) = \log a - \log b$

$\text{Log}_a b = \dfrac{\log_{10} b}{\log_{10} a}$

Practice Exercise 3.2

3. Fill in the blank spaces:

(i) $\log_a((pq)r\ldots)$ = _____

(ii) $\log_a(p/q)$ = _____

(iii) $\log_a(p^q)$ = _____

(iv) $\log_q p$ = $\dfrac{?}{\log_a q}$

4. Solve the following:

(i) Find 'x' if $\log_x\left(\dfrac{9}{16}\right) = -2;\ x>0$ _____

(ii) $\log_{0.1}1000$ _____

(iii) $\log\left(\dfrac{a}{b}\right)+\log\left(\dfrac{b}{c}\right)+\log\left(\dfrac{c}{a}\right)$ _____

(iv) $\log\left(\dfrac{5}{7}\right)+\log\left(\dfrac{28}{25}\right)+\log\left(\dfrac{5}{4}\right)$ _____

(v) $\log_6 16.\ \log_4 216 = a$ _____

(vi) $\dfrac{1}{\log_a ab}+\dfrac{1}{\log_b ab}$ _____

(vii) Find 'x' if $\dfrac{2}{3}\log x + \log 5 = \log 80$ _____

(viii) Find 'x' if $2\log x + \log 5 - \log 3 = \log 15$ _____

(ix) Find the values for 'x' if $\log_2(x+y) = 4$ & $\log_{x-y} 16 = 2$

_____

3.3 Functions

Concept of function:

Y = f(x) means that 'y' is a function of 'x' such that for every value of 'x' there exists a value of 'y'.

The value of a function y = f(x) for x = a is obtained

by substituting 'a' for 'x' in f(x) and is denoted by f(y).

For e.g.: If $f(x) = x^3+3$ find f(2)

$\therefore f(2) = 2^3+3 = 11$

$\therefore f(2) = 11$

Practice Exercise 3.3

5. Solve:

(i) If $f(x) = 7x^3 - 13$, find f(2). _____

(ii) If $f(x) = 6x^2-3x+2$, find f(x) when x = -1. _____

(iii) If $f(x) = 2x^2- \frac{1}{x^2}$, find $f\left(-\frac{1}{x}\right)$ _____

(iv) If $f(\theta) = \sin\theta + \cos\theta$; find $f(-\theta)$. _____

(v) If $(x) = 2x^2 - 3x + 7$; Find: $[f(x)-\{f(x+1)-3f(1)\}]$. _____

(vi) If $f(x) = \log x^{\frac{1}{x}}$, and $g(x) = e^x$ find f[g(x)]. _____

(vii) If $f(x) = \frac{x-1}{x+1}$, find $f\left(\frac{1}{x}\right)/\left[\frac{1}{f(x)}\right]$, if $x \neq 0$. _____

Odd and Even function:

A function is said to be odd if

$f(-x) = -f(x)$

For e.g.: $f(x) = 2x^5-3x^3$

A function is said to be an even function if

$f(-x) = f(x)$

For e.g. $f(x) = x^2$

6. Identify even/odd functions in the following:

(i) $f(x) = ax^4+bx^2+c$ _____

(ii) $f(x) = \frac{x-x^{-1}}{x}$ _____

(iii) $f(x) = \frac{2x-x^{-1}}{5}$ _____

7. Find the minimum value of:

$f(x)=x^2-x$ _____

Answer Key

Practice Exercise 1.1

1. 0 2. 5 3. 4 4. 5 5. 5

Practice Exercise 1.2

1. 11 2. 203/32 3. -52 4. 5/7 5. 25/12 6. 52/25
7. 15 8. 6 9. 5.623 10. 66/7

Practice Exercise 1.3

1. 50 2. 10,40 3. 35,15 4. 4, 24 5. 50 , 20

6. 9 7. 63 , 70 , 77 , 84 8. 338 9. 180

10. 80 11. 19 , 20 , 21 12. 1635 , 270 13. 5 , 3 14. 500 * 250

15. 10m , 7m

Practice Exercise 2.1 and 2.2

1. -4 , -1 2. -6 , -2 3. 12 , -3 4. 18 , -3 5. -21 , 6

6. -9 , 2 7. -6 , 3 8. 5 , -2 9. 8 , -3 10. 12 , -10

Practice Exercise 2.3

1. 8 x 15, 10 x 12 2.73 3.17 4.10 x 10

5.19 6. 8 7.3 8.6 9.28 x 3, 21 x 4

10.11 11.44 12.8

Practice Exercise 3.1

1. i) 6log 2 ii) 3log 3 iii) 4log 2 iv) ylog a

2. i) A^A ii) Y^a iii) 5^3 iv) 9^2

v) 4^3

Practice Exercise 3.2

3. i) $\log_a p + \log_a q + \log_a r$ .... ii) $\log_a p - \log_a q$
iii) $q \log_a p$ iv) $\log_a p$

4. i) 4/3 ii) -3 iii) 1 iv) 1 v) 6 vi) 1

vii) 64 viii) 3 ix) 10

Practice Exercise 3.3

5. i) 42 ii) 11 iii) $2/x^2 - x^2$ iv) $\cos\theta - \sin\theta$ v) -4x + 19

vi) 1 vii) $-(x-1)^2/(x+1)^2$

6. i) even ii) even iii) odd

7. 0