

MATH Workbook

Foreword

One of the prime objectives of education is to develop thinking skill in learners.

Thinking skills is essential to success in education, career and life in general.

Mathematical reasoning is one of the essential skills to be an effective critical thinker.

An individual who is adept at mathematical reasoning is able

- **to read between the numbers to deduce patterns**
- **to relate various parameters and arrive at a relationship specific to the problem**
- **to construct alternate scenarios with the same parameters thereby resulting in multiple solutions**
- **to correlate a known or an analogous formula/theorem to the given problem**
- **to converge at a solution using different approaches**
- **to convert a problem from the given form into another less complex form**

We are convinced that the preparation towards the complex area of mathematical reasoning should comprise more than referring to discussion of sample questions.

Therefore this module attempts to present before you learning experiences which will empower you with essential mathematical reasoning and problem solving tools.

This module is presented in a format which aims to create a virtual teacher who would hand hold you while you explore the realm of math. The concepts are explained in a lucid manner with minimalist words and maximum transfer learning.

It takes more than one reading to fully assimilate and appreciate the concepts.

While solving problems, initially use the pen and paper extensively. After you learn computational techniques try to 'visual think' the computation.

Remember math learning is not about remembering formula or tables or theorems, it is all about learning the processes involved in successful problem solving.

Enjoy learning

1. Number System and averages

There are different terms in number system. Natural integers, fractions, mixed fractions, prime numbers, composite numbers, imaginary numbers, real numbers, rational numbers, irrational numbers, decimals, odd numbers, even numbers, co-prime numbers are some of the commonly used terms in the number system.

1.1 Operations on a fractions

Fractions can be multiplied, added, divided and subtracted

To add fractions or to subtract them there are two methods

Method 1:

Make sure the denominators are the same number

To add $\frac{4}{5}$ and $\frac{6}{50}$

Convert $\frac{4}{5}$ to a fraction with denominator 50

This can be done by multiplying 4 and 5 by 10

$$\text{Hence } \frac{4}{5} = \frac{4 \times 10}{5 \times 10} = \frac{40}{50}$$

$$\text{Therefore } \frac{4}{5} + \frac{6}{50} = \frac{40}{50} + \frac{6}{50} = \frac{46}{50}$$

Method 2:

Find LCM of the denominators

Subtract $\frac{51}{8} - \frac{16}{6}$

LCM of the denominators(8,6) is 24

Multiply each fraction with a number such that the denominator = 24

$$\frac{51}{8} = \frac{51 \times 3}{8 \times 3} = \frac{153}{24}$$

$$\frac{16}{6} = \frac{16 \times 4}{6 \times 4} = \frac{84}{24}$$

$$\frac{51}{8} - \frac{16}{6} = \frac{153}{24} - \frac{84}{24} = \frac{69}{24}$$

Practice Exercise 1.1

Fill in the space with the correct answer

1. $\frac{5}{2} + \frac{7}{3} =$ _____

2. $\frac{9}{4} + \frac{7}{5} =$ _____

3. $3\frac{1}{3} + 4\frac{2}{5} =$ _____

4. $4\frac{5}{16} - 2\frac{1}{8} =$ _____

5. $2\frac{3}{16} - 1\frac{3}{8} =$ _____

6. $\frac{51}{8} + \frac{16}{6} =$ _____

Fractions can be multiplied by multiplying the denominators and numerators separately

Practice Exercise 1.2

Fill in the space with the correct answer

1. $\frac{5}{2} \times \frac{7}{3} =$ -----

2. $\frac{9}{4} \times \frac{7}{5} =$ -----

3. $3\frac{1}{3} \times 4\frac{2}{5} =$ -----

4. $4\frac{5}{16} \times 2\frac{1}{8} =$ -----

5. $2\frac{3}{16} \times 1\frac{3}{8} =$ -----

Fractions can be divided by multiplying the reciprocal of one fraction with the other fraction

$$\frac{5}{2} \div \frac{7}{3} = \frac{5}{2} \times \frac{3}{7} = \frac{15}{14}$$

Invert $\frac{7}{3}$ to $\frac{3}{7}$ and multiply with $\frac{5}{2}$

Practice Exercise 1.3

Fill in the space with the correct answer

1. $\frac{9}{4} \div \frac{7}{5} =$ -----

2. $3\frac{1}{3} \div 4\frac{2}{5} =$ -----

$$3. 4\frac{5}{16} \div 2\frac{1}{8} = \text{-----}$$

$$4. 2\frac{3}{16} \div 1\frac{3}{8} = \text{-----}$$

$$5. \frac{51}{8} \div \frac{16}{6} = \text{-----}$$

1.2 Method to determine the square root of a given number

Observe the method to find the square roots of 23456

1. Start from Left, Segregate two numbers at a time.
i.e 2,34,56

2. Find the square of a number which is lesser than the left most number(i.e. 2) . Place the number(1) to the left of the line. Place the square below the left most number.

$$\begin{array}{r} \bullet \quad \bullet \quad \times \\ \bullet \quad \bullet \quad \times \\ \times \quad \times \end{array} \begin{array}{r} \hline 2,34,56 \\ 1 \\ \hline 1 \quad 34 \end{array}$$

3. Subtract the two numbers.

4. Bring the next two numbers down.

5. Double the number which is placed above the line (1) and place on the left of the number which is brought down.

$$\begin{array}{r} \hline 1 \quad 5 \\ \hline 1 \quad 2,34,56 \\ 1 \\ \hline 25 \quad 1 \quad 34 \\ 1 \quad 25 \\ \hline 30y \quad 9 \quad 56 \\ \cdot \quad y \end{array}$$

6. Find the value of x such that (2 x) × (x) gives a number equal to or less than 134. (i.e. 23 x 3 or 24 x 4 or 25 x 5 or 26 x 6....)

7. Since 25 x 5 is nearest to 134. Replace x as 5 & subtract.

$$\begin{array}{r}
 \bullet \quad 59 \\
 \bullet \quad \overline{2,34,56} \\
 \quad \quad 1 \downarrow \\
 25 \quad \overline{134} \quad \downarrow \\
 \quad \quad 125 \\
 303 \quad \overline{956} \\
 \times 3 \quad \overline{909} \\
 \quad \quad \quad 47
 \end{array}$$

8. Double the number above the top line (15) and write it on the left region (30y)

9. Determine the value of y $(30y) \times (y)$ which gives a value lesser than or equal to 956.

10. Replace y as 9.

Nearest Square Root of 23,456 = 159.

Practise Exercise 1.4

Enter the nearest square root of the given number in the space provided

1. 2345 -----
2. 5684 -----
3. 430 -----

1.3 Method to test whether given number is prime

A prime number is a number which can be divided by itself and one only.

Eg. 2,3,5,7,11,13...

1 is not a prime number, 2 is the only even prime number.

To determine whether the given number is a prime number or not divide the given number by all the prime numbers starting from 2 till the square root of the given number. If it is divisible by any of the numbers then the given number is not a prime number.

State whether the given number is prime or not prime in the space provided

Practice Exercise 1.5

1. 567 -----
2. 787 -----
3. 431 -----

1.4 Method to find LCM of given numbers

LCM is the least common multiple of given set of numbers

Find LCM of 48, 24, 36

Divide successively by numbers which divides atleast one of the numbers given

2 divides 48,24 and 36

2	48, 24,36
---	-----------

Now 3 divides 12,6,and 9

2	24, 12,18
---	-----------

Now 2 divides 4 and 2 and 3 divides 3

3	12, 6,09
---	----------

Divide the numbers by suitable numbers

3	4, 2, 03
---	----------

2	4, 2, 1
---	---------

2	2, 1, 1
---	---------

	1, 1, 1
--	---------

Until they get simplified to 1 each

Multiply all the numbers which divided 48,24,36

LCM = $2 \times 2 \times 3 \times 3 \times 2 \times 2 = 144$

Practice Exercise 1.6

Find LCM of the following numbers

1. 25, 10 and 8 =

2. 34, 7, 9 =

3. 18, 24, 48 =

4. 25, 30, 50 =

1.5 Compare fractions

Fractions can be compared by

1. Making the numerator similar

2. Making the denominator similar

By making numerator similar

Is $\frac{5}{7}$ greater than $\frac{11}{13}$

To compare these fractions

Method 1: Make numerator similar

Determine LCM of 5 and 11

LCM = 55

Multiply $\frac{5}{7}$ by 11 to make numerator 55

$$\frac{5}{7} = \frac{5 \times 11}{7 \times 11} = \frac{55}{77}. \text{ Similarly}$$

$$\frac{11}{13} = \frac{11 \times 5}{13 \times 5} = \frac{55}{65}$$

The numerators of both fractions are same(55).

If the numerators are same, smaller the denominator greater the fraction

$$\text{Hence } \frac{55}{77} < \frac{55}{65} \text{ Therefore } \frac{5}{7} < \frac{11}{13}$$

Method 2: Make denominators similar

Determine LCM of 7 and 13

$$\text{LCM} = 91$$

Multiply $\frac{5}{7}$ by 13 to make numerator 91

$$\frac{5}{7} = \frac{5 \times 13}{7 \times 13} = \frac{65}{91} \text{ . Similarly}$$

$$\frac{11}{13} = \frac{11 \times 7}{13 \times 7} = \frac{77}{91}$$

The denominators of both fractions are same(91).

If the denominators are same, greater the numerator greater the fraction

$$\text{Hence } \frac{65}{91} < \frac{77}{91} \text{ . Therefore } \frac{5}{7} < \frac{11}{13}$$

Practice Exercise 1.7

Enter the following fractions in ascending order

$$\frac{3}{2}, \frac{5}{13}, \frac{6}{55}, \frac{17}{66} = \text{-----}$$

$$\frac{2}{3}, \frac{4}{9}, \frac{5}{6}, \frac{11}{12} = \text{-----}$$

Practice Exercise 1.8

Enter the following fractions in descending order

$$7\frac{3}{23}, \frac{51}{46}, \frac{1}{69} = \text{-----}$$

$$\frac{6}{13}, 4\frac{3}{35}, 2\frac{5}{7}, 1\frac{1}{5} = \text{-----}$$

1.6 Remainders

The remainder can be determined using the formulae

$$\text{Dividend} = \text{Divisor} \times \text{quotient} + \text{remainder}$$

Find the remainder when 29 is divided by 3

Here dividend = 29

Divisor = 3

Hence

$$29 = 3 \times 9 + 2$$

Where 9 is the quotient and 2 is the remainder

What is the least number that must be subtracted to 29 to make it perfectly divisible by 3

Write 29 in terms of 3

As per the formula: Dividend = Divisor x quotient + remainder

$$29 = 3 \times 9 + 2$$

To make the number perfectly divisible the remainder should be 0

Hence subtract 2 on both sides

$$29 - 2 = 3 \times 9 + 2 - 2$$

$$27 = 3 \times 9 + 0$$

Hence 2 must be subtracted to make 29 perfectly divisible by 3

Practice Exercise 1.9

Fill in the answers

1. The quotient arising from the division of 1139 by a certain number is 66 and the remainder is 13. Find the divisor

2. What is the least number that must be subtracted from 8975 to make it perfectly divisible by 7

3. What is the least number that must be subtracted from 7893 to make it perfectly divisible by 8

4. What is the least number that must be added to 7893 to make it perfectly divisible by 7.

5. What is the smallest 4 digit number that is perfectly divisible by 7

6. What is the greatest 4 digit number that is perfectly divisible by 8

7. A perfect square number $\{200 < n < 300\}$ when divided by 8 gives a remainder 1. What is the remainder when the same number is divisible by 7?

8. How many distinct prime factors are there for $24^3 \times 20^4 \times 13^9$

9. What is the last digit of $2412 \times 20986 \times 1334543$

10. Find the remainder of $243 \times 209 \times 133$ when it is divided by 7

1.7 Averages

Averages = Sum of the numbers / number of numbers

Practice Exercise 1.10

1. In an examination, the average score of 60 students is 45. If each student gets 10 more marks, then what is the new average?

2. In an examination, the average score of 60 students is 45. The average of the remaining 40 students is 60. Find the average of the entire class?

3. The average weight of 10 balls is 20 gms. If the heaviest ball which weighs 30gms is removed and a new ball which weighs 50 gms is added, then what is the new average?

4. The average weight of 8 people increases by 10 kgs when one person who weighs 60 kgs is replaced by another man. Find the weight of the other man.

5. I have ten pounds of \$10 type of wheat and I have 20 pounds of \$ 5 type of wheat. If mix them what will be my average price per pound of wheat.

2. Percentages

The word percentages mean per hundred. It is denoted by “%”

2.1 Percent number

$$80\% \text{ of } 100 = \frac{80}{100} \times 100 = 80$$

$$56\% \text{ of } 200 = \frac{56}{100} \times 200 = 112$$

Practice Exercise 2.1

Solve

1. 4.5% of 30 =

2. 67% of 300 =

3. 87 % of 500 =

4. $\frac{1}{2}$ % of 50 =

Convert into percentages

Multiply the number with 100%

2. 3 = 2.3 x 100% = 230%

$$1\frac{1}{2} = \frac{3}{2} \times 100 \% = 150\%$$

Practice Exercise 2.2

Convert into percentages

1. 5 =

2. $1\frac{6}{7}$ =

3. $\frac{2}{7}$ =

2.2 Relative percentage

One number can be expressed in terms of another using relative percentage.

If A is 30% more than B then $A = (100 + 30) \% \text{ of } B = 1.3B$

If A is 40% less than B then $A = (100 - 40) \% \text{ of } B = 0.6 B$

Percentage change = $\frac{\text{Final value} - \text{initial value}}{\text{initial value}} \times 100$

If in 2008 the sales was \$20000 and in 2009 the sales was \$34000

then the percentage change = $\frac{34000 - 20000}{20000} \times 100 = 85\%$

Practice Exercise 2.3

Solve and enter the answer

1. If A is 20% more than B and B is 30% more than C, then write A in terms of C?

2. If A is 20% more than B, then how much lower is B's score as compared to A's ?

3. If A scored 30% of the total mark and if B scored 10% more than A, then what is B score in terms of the total score

4. If A's salary is \$ 5000, he spent 10% on groceries and 40% on rent. How much has he saved so far?

5. If B spent 15% of his salary on groceries and 45% on rent and was left with \$ 300. How much was his salary originally

6. Out of 500 students in school 90% of them are from city A others are from city B. 40% of students from city A and 50% of the students from city B come to school in a bicycle

- (i) How many students come in a bicycle to school? -----
- (ii) How many students do not come to school in a bicycle? -----
- (iii) How many students from city B do not come in bicycle? -----

7. Observe this sales table and answer the questions that follow

Company	2006	2007	2008	2009
A	45	87	34	85
B	36	35	23	43
C	67	26	22	37
D	24	12	25	90

(i). What is the maximum percentage change in sales in any year over the previous year's data?

(ii). Sales of Company A in 2007 was how much percentage less than sales of company D in 2009?

(iii). What is the difference of combined sales of company A and B over that of C from 2007 to 2009?

8. If the population of city A grows 10% annually, then how much percentage greater is the population in 2008 than the population in 2006.

9. If x increases by 20% then by how much percentage does x^2 increase?

10. If the side of a square increases by 33.3 % then by how much percentage does the area increase ?

11. At a certain point during a baseball tournament, the Red Sox team calculated that they had won 60% of the matches they had played. They devised a new strategy and won all the six remaining matches. Now their win percentage was 75%. How many games did they play in all?

3. Simple interest and compound interest

Simple interest is given by the formula: $SI = \frac{pnr}{100}$

Amount = SI + P

Compound interest is given by the formula: $CI = p\left(1 + \frac{r}{100}\right)^n - p$

Amount = SI + P

where P = principal

N = number of years

R = rate percent

Amount = SI + P

3.1 Problems associated with SI

Exercise 3.1

Find SI

1. \$ 4000 for 5 years at 5% per annum(PA)

2. \$ 3560 for 4 years at $6\frac{1}{4}$ % P.A

3. \$ 250 for 24 months at 10% P.A

4. \$ 5000 for 24 months at 10% per month

5. \$ 5000 for 5 years at 10% per month

6. \$ 50 for 146 days at 6% P.A

Find rate(r)

7. \$ 506 amounts to \$ 1460 in 6 years

8. A sum of money doubles in 10 years

9. The difference in SI on \$ 1650 and \$1800 in 8 years is \$ 30

Find number of years(n)

10. In how many years will \$5000 amount to \$7000 at 4% pa

11. In how many years will a sum of money double itself at 5% pa

12. The SI on a loan will be \$600 in 10 years, If the principal is trebled in 5 years what will be the total interest at the end of the 8th year

3.2 Problems associated with CI

Practice Exercise 3.2

Find CI

1. \$4000 in 3 years at 10% pa

2. \$40 in 2 years at 10% per quarter

Find principal

3. Amount = \$400 in 2 years at 10% pa

4. Amount = \$300 in 2 years at 10% half yearly

Find rate percent

5. \$400 amounts to \$1600 in 2 years

6. The sum doubles itself in 4 years

7. Calculate the CI at the end of two years for \$4000 when the rate percent is 10% for the first year and 20% for the second year.

3.3 General problems

Practice Exercise 3.3

1. Calculate the CI for a principal which gives a SI of \$4000 at the end of 2 years at 5% pa

2. Calculate the principal if the difference of the CI and SI is \$5 for a period of 2 years at 5% pa

3. Calculate the difference in CI and SI for \$5000 for a period of 3 years at 10% pa

4. I purchased a car worth \$25000. I make a down payment of \$5000. I'm going to pay the remaining amount through annual instalments at the rate of 10% CI for 3 years. What is the total interest I will be paying for the car?

4. Profit loss and discount

Profit = selling price – cost price

Profit percentage = $\frac{\text{selling price} - \text{cost price}}{\text{cost price}} \times 100$

Loss percentage = $\frac{\text{cost price} - \text{selling price}}{\text{cost price}} \times 100$

Practice Exercise 4.1

Find profit/loss percentage

1. Selling price is \$25000 and cost price is \$ 20000.

2. Selling price is 3 times the cost price

3. By selling 100 mangoes the seller gains the selling price of 20 mangoes

4. Two cars sold at \$10,000 each. One sold at a profit of 10% and the other sold at a loss of 10%

5. Pens are bought at 4 for \$1 and sold at 3 for \$2

Selling price = cost price + profit % of cost price

Selling price = cost price - loss % of cost price

6. What will be the cost price of an article if the selling price is \$5000 and if it is sold at a loss of 10%

7. A man bought 5 eggs for a rupee. If he wanted to make a profit of 20%, how many eggs per rupee should he sell?

8. A sells an article to B at a profit of 10%. B sells it to C at a loss of 20% and C sells it to D at a profit of 10%. If A bought the article for \$1000, then how much did D buy it for?

9. Henry buys 10kgs of type 1 wheat at \$5 per kilo and buys 20kgs of type 2 wheat at \$10 per kilo. He mixes them and sells them at \$9 per kilo. What is his profit percentage?

10. A shopkeeper sells an item at a profit of 10%. He uses a balance which reads 900gm for 1kg. How much profit does he actually make?

Marked price = Selling price - discount % of Selling price

11. The marked price of an article is 25% above the cost price. What will be the gain after allowing a 20% discount to the customer.

12. Oranges are bought at 12 for \$5 and sold at 5 for \$12. What is the loss or profit percent

13. One third of good in the gowdown are sold at a loss of 20%. At what profit percentage should the remaining goods be sold such that there is a overall profit of 10%. -----

14. A shopkeeper gives a discount of 10%. He further gives a discount of 15%. How much discount does he give overall.

15. A man buys an article for \$20 after getting a discount of 10%. How much was the shop keeper quoting.

16. What is the equivalent single discount of two successive discounts of 5% and 15%.

5. Ratio and proportion

Ratio is the relation of one quantity with respect to another. The ratio of two quantities x and y is given by x:y. A ratio is a pure number and doesn't have any units.

5.1 Problems associated with ratios

Express the following ratios in their simplest forms:

$$18:40 = 9 : 20$$

$$24 : 88 = 12 : 44 = 6 : 22 = 3 : 11$$

Exercise 5.1

1. $40 : 25 =$ -----

2. $30 : 66 =$ -----

3. $930 : 360 =$ -----

4. $1000 : 500 =$ -----

5. $560 : 343 =$ -----

Which ratio is the biggest?

6. $2 : 7$ & $3:8$ -----

7. $3:7$ & $5:12$ -----

8. $1034 : 567$ & $2340 : 690$ -----

9. Represent the ratio

i. 3 : 4 with denominator 20 -----

ii. 5 : 6 with numerator 30 -----

iii. 7 : 4 with numerator 42 -----

iv. 9 : 4 with denominator 100 -----

v. 13 : 4 with numerator 42 -----

5.2 Compare ratios

When ratios are compared they must be represented in the same units.

2 hrs 30 mins : 3hrs 10 mins can be represented in minutes as

2hrs 30 mins = 150mins and 3hrs 10 mins = 190 mins

Required ratio = 150 : 190 = 15 : 19

Exercise 5.2

Simplify the following ratios

1. 2 hours 30 minutes and 5 hours 50 minutes = -----

2. 3 years 4 months and 4 years 11 months = -----

3. 3 meters 40 cm and 4 meters 10 decimeters = -----

4. 400 gms and 8.8 kgs = -----

5. 3 feet 12 inches and 4 feet 10 inches = -----

To compare ratios their numerator or denominators must be same

Which ratio is bigger 3 : 4 or 6 : 9 ?

Make the numerators common : 3 : 4 = 3 x 2 = 4 x 2 = 6 : 8

Now compare the ratios 6 : 8 or 6 : 9

Obviously 6: 8 is bigger

Hence 3 : 4 is bigger than 6 : 9

Which is bigger?

6. 7 : 40 or 5 : 30 -----

7. 2 : 30 or 6 : 15 -----

8. 6.25 : 4.2 or 5 : 2.25 -----

9. 2.5 : 7/ 2 or 3 : 4 -----

10. 11 : 2 or 3 : 10 -----

5.3 Converting ratios into numbers

Ratio per se doesn't represent the actual quantities of elements.

To convert the ratio into quantities, introduce a constant (say k) to convert the ratio into quantities.

If A and B split a \$900 in the ratio 4:5, then the amount A gets = $4k$ and the amount B gets = $5k$ (k is a constant)

Hence $4k + 5k = 900$

$k = 100$

A gets $4k = 4 \times 100 = 400$ and B gets $5k = 5 \times 100 = 500$

Exercise 5.3

Answer the following questions

1. Two numbers whose sum is 120 are in the ratio 1:4. Find the numbers.

2. Two numbers whose product is 1600 are in the ratio 1:4. Find the numbers. -----

3. **Two numbers are in ratio 5 : 7. If 2 is added to denominator the ratio becomes 2 : 3. Find the number**

4. **\$5400 is divided in the ratio 3: 4 : 5 : 6 among four people, How much does each person get** -----

5. **A certain amount is divided in the ratio 3: 4 : 2 : 9 among four people, If the 2nd person gets \$400 more than the 3rd person then what is the total amount distributed to the 4 people**

6. **What is the ratio of the area of a square to the area of the circle if the side of the square is equal to the radius of the circle.**

7. **If $\frac{x}{3} = \frac{5}{7}$ then $\frac{4+x}{5-x} =$** -----

8. **A purse contains coins consisting of denomination \$1, \$5, \$10. If the number of coins are in ratio 2 : 3 : 8 and the total value is \$72. Find the number of coins of each kind.**

9. **Two numbers are in ratio 7 : 5. The sum of two numbers is 108. If 1 is added to the numerator and 5 subtracted from the denominator, then the ratio is**

10. **Two numbers are in the ratio 5 : 11. If 1 is added to the first and 5 is subtracted from the second, then the new ratio is 1 : 2. Find the numbers.**

5.4 Proportion

The ratio of two ratios is called proportion. If a,b,c,d are in proportion they

are represented as $a : b :: c : d$. r $\frac{a}{b} = \frac{c}{d}$

Exercise 5.4

Solve for x

$$x : 4 :: 27 : 36$$

$$\frac{x}{4} = \frac{27}{36}, x = \frac{27}{36} \cdot 4 = 3$$

1. $4 : x = 60 : 20$ -----

2. $5 : x = 40 : 65$ -----

3. $6 : 30 = 8 : x$ -----

4. $5 : 28 = 40 : x$ -----

5. $5 : 28 = x : 40$ -----

6. $3 : 7 = 2 : x$ -----

5.5 General problems

Exercise 5.5

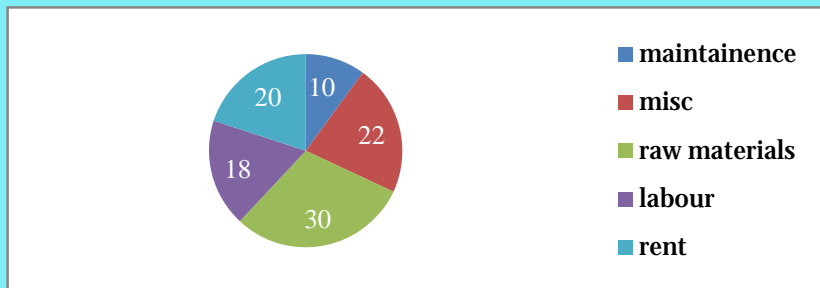
1. If cost price of 20 apples is \$35. What is the cost of 30 apples?

2. B eats twice as many apples as A eats. If A eats 10 apples then B eats how many apples?

3. 8 men or 16 boys can do a piece of work, If 32 boys complete a work in 10 days then how many men can complete the work in 10 days?

4. The pie chart gives the percentage allocation for a firm

All numerals are in percentages



i. What is the angle of the sector representing misc expenses -----

ii. What is the ratio of expenditure on raw material to labour -----

iii. What is the ratio of expenditure on maintenance to expenditure on labour -----

iv. If total expense for the firm was \$5000, what is the expense on rent -----

5. A vessel contains 56 liters of a mixture of water of milk in the ratio 5:2. How many liters of water must be added to it so the ratio of milk and water becomes 4:5?

6. A 18L wine bottle has wine and water in the ratio 5:4. If 10 liters of water is added what is the new ratio of wine and water

7. Two numbers are in the ratio 8 : 5, if 9 is added to each they will be in ratio 11 : 8. Find the two numbers.

6. Time speed and distance

Time, speed and distance are related with each other.

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

If speed increases, time decreases. (distance remains constant)

Exercise 6.1

Solve these problems

1. What is the time taken to travel 20 kms at a speed of 10 kmph?

2. What is the distance travelled by a car in 2hrs if it travels at a rate of 40mph?

3. A certain distance is covered in 1 hr at 50kmph. What is the time taken to cover double the distance at 20kmph?

4. A car travels a distance of 20kms in 2hrs and travels triple the distance at a speed of 30kmph. What is the time taken for the entire journey?

5. A bus travels 250kms, 100kms and 50kms at a speed of 10kmph, 25kmph and 20 kmph. What is the average speed of the bus? (average speed = total distance/total time)

6.2 Conversion of units

$$1 \text{ minute} = 60 \text{ seconds}$$

$$1 \text{ hour} = 60 \text{ minutes} = 3,600 \text{ seconds}$$

$$1 \text{ metre} = 100 \text{ centimetre} = 1,000 \text{ millimetre}$$

$$1 \text{ kilometre} = 1,000 \text{ metre} = 1,000 \times 1,000 \text{ millimetre}$$

Convert kmph into m/sec

$$\begin{aligned} 18 \text{ kmph} &= \frac{18 \times 1 \text{ km}}{1 \text{ hr}} = \frac{18 \times 1000 \text{ m}}{60 \times 60 \text{ sec}} \\ &= 18 \times \left(\frac{5}{18} \right) \text{ m/sec} = 5 \text{ m/sec} \end{aligned}$$

Exercise 6.2

Convert the following into m/sec

1. 54 kmph = ----- m/sec

2. 180 kmph = ----- m/sec

3. 450 kmph = ----- m/sec

4. 540 kmph = ----- m/sec

Convert m/sec into kmph

$$5 \text{ mps} = \frac{5 \times 1 \text{ m}}{1 \text{ sec}} = \frac{5 \times \frac{1}{1000} \text{ km}}{\frac{1}{3600} \text{ sec}} = 5 \times \frac{18}{5} = 18 \text{ kmph}$$

Convert the following into kmph

5. 18 m/sec = ----- m/sec

6. 55 m/sec = ----- m/sec

7. 60 m/sec = ----- m/sec

8. 42 m/sec = ----- m/sec

6.3 Relative speed

If two bodies move in same direction at A kmph and B kmph, then the relative speed = $A + B$ kmph

If two bodies move in opposite direction at A kmph and B kmph, then the relative speed = $A - B$ kmph

Exercise 6.3

A starts walking from point X to point Y at a speed of 10 kmph. B starts walking from point Y to point X at a speed of 20 kmph. If distance between A and B is 300 km.

1. After two hrs what is the difference between A's position and B's position?

2. When will they be 150 kms apart?

3. After 4 hrs, how many kms will A be from X?

4. After 8 hrs, how many kms will B from X?

5. When will they meet?

6. At what distance from X will they meet?

A starts walking from point X to point Y at a speed of 10kmph. B also starts walking from point X to point Y at a speed of 5kmph. If distance between A and B is 300km.

7. After two hrs what is the difference between A's position and B's position?

8. When will they be 50kms apart?

9. After 4 hrs, how many kms will A be from B's position?

A starts walking from point X to point Y at a speed of 10kmph. 2 hrs later B also starts walking from point X to point Y at a speed of 15kmph. If distance between A and B is 300km.

10. Where is A located from X, when B starts?

11. After two hrs what is the difference between A's position and B's position?

12. When will B overtake A?

13. After 4 hrs, how many kms will A be from B's position?

14. When will reach A reach point Y.

15. At what distance from Y will B overtake A?

A and B walk around a park. The circumference of the park is 20km. Both A and B start from the entrance. A travels clockwise and B travels anticlockwise.

If A travels at a speed of 4kmph and B travels at 8kmph, then

16. In how many hours will A complete one round?

17. When B completes one round, B will meet A how many times?

18. In 4 hrs B will meet A how many times?

19. In how many hours will they meet for the first time?

20. After walking for 2 hrs, B stops and remains stationary. In how many hours will A take to complete the same distance that B completed?

6.4 Effect of two forces

The movement of one body is influenced by the force acting on that body.

A ball when thrown in the breeze will move at a lesser speed than when it moves when there is no breeze.

A ball when thrown with the breeze will move at a higher speed than when it moves when there is no breeze.

A boat moving in a river will move at a higher speed if it flows along the river

A boat moving in a river will move at a lower speed if it flows against the river

If force A aids the motion of force B then total speed = speed of A + speed of B

If force B opposes the motion of force B then total speed = speed of A - speed

of B or speed of B - speed of A.

Exercise 6.4

1. What is the effective speed of a ball moving at 30m/sec when it is thrown against the breeze whose speed is 2m/sec?

2. What is the effective speed of a ball moving at 100m/sec when it is thrown in the direction of the breeze whose speed is 2m/sec?

3. A boat can travel at a speed of 8kmph.If it moves downstream on a river, in how many hours will it travel a distance of 40kms.(assume rivers speed – 2kmph)

4. A man travels up on a moving up escalator at a speed of 10 steps per minute. The escalator moves at a speed of 1 step per minute. If the man travels for 1 min, how many steps will the man climb if the escalator is stationary?

7. Time and work

A does work alone for 5 days. B does the work alone for 6 days.

Quantity of work done by A in one day = $\frac{1}{5}$.

Quantity of work done by B in one day = $\frac{1}{6}$.

Quantity of work done by A and B together = $\frac{1}{5} + \frac{1}{6} = \frac{11}{60}$

Number of days required = $60/11$ days as work done by both of them individually is $11/60$

Exercise 7.1

1. A can do a work in 5 days. B can complete the same work in 10 days. How many days do they take to complete the work together?

2. A and B can do the work together in 12 days. A alone can complete the work in 21 days. How many days does B take to do work alone?

3. A can do the work alone in 10 days. In 5 days what ratio of the work is completed?

4. A can do the work alone in 10 days. B can do the same work independently in 25 days. If A works independently for 4 days and B works independently for 6 days. What ratio of the work is left incomplete?

5. A can do a work in 20 days. B can do it in 25 days. They work together for 5 days and then A leaves. In how many days will B take to complete the work

alone

6. A can do $\frac{1}{6}$ th of the work in 5 days. In how many days will A complete the work?

7. A can do $\frac{1}{5}$ th the work in 5 days. B can do $\frac{1}{4}$ th the work in 5 days. If both of them work together, then in how long will they take to complete the work.

8. A,B and C completes a work in 50 ,80, 100 days respectively.In how many days will

(i)A and B take to complete the work -----

(ii) B and C take to complete the work -----

9. A,B and C can complete a work in 25 days. If A and B can complete the work in 30 days, in how many days will C work alone to complete the work?

10.A and B can complete a work in 20 days. B and C can complete the work in 30 days and A and C can complete the work in 40 days. In How many days will C work alone to complete the work ?

Answer Key

Practice Exercise 1.1

1. 29/6 2. 73/20 3. 116/15 4. 35/16 5. 13/16
6. 217/24

Practice Exercise 1.2

1. 35/6 2. 63/20 3. 44/3 4. 1173/128 5. 385/128

Practice Exercise 1.3

1. 45/28 2. 25/33 3. 69/34 4. 35/22 5. 153/64

Practice Exercise 1.4

1. 48 2. 75 3. 21

Practice Exercise 1.5

1. No 2. Yes 3. Yes

Practice Exercise 1.6

1. 200 2. 2142 3. 144 4. 150

Practice Exercise 1.7

1. $6/55 < 17/66 < 5/13 < 3/2$
2. $4/9 < 2/3 < 5/6 < 11/12$

Practice Exercise 1.8

1. $7^{3/23} > 51/46 > 1/69$
2. $4^{33/35} > 2^{5/7} > 1^{1/5} > 6/13$

Practice Exercise 1.9

1. 17 2. 1 3. 5 4. 3 5. 1001 6. 9992
7. 1 & 2 8. 4 9. 6 10. Zero

Practice Exercise 1.10

1. 73.33 2. 51 3. 22 4. 140 kg 5. 6.67 \$/pound

Practice Exercise 2.1

1. 1.35 2. 201 3. 435 4. 0.25

Practice Exercise 2.2

1. 500 % 2. 185.71% 3. 28.57 %

Practice Exercise 2.3

1. $A = 1.56 C$ 2. $B = 0.83 A$ (17% Lower)
3. 33% 4. 2500 \$
5. 750\$
6. i) 205 ii) 295 iii) 25
7. i) 260% in D in year 2009 ii) 3.33% iii) 222
8. 21 % 9. 44 % 10. 77.76% 11. 16

Practice Exercise 3.1

1. 1000 2. 890 3. 50 4. 1000 5. 30, 000
6. 1.2 7. 48.08% 8. 20% 9. 2.5 10. 35
11. 40 12. 480 \$

Practice Exercise 3.2

1. 1324 2. 45.74 3. 330.58 4. 204.90 5. 100%
6. 18.9 % 7. 1280

Practice Exercise 3.3

1. 4100\$ 2. 2000 3. 155 4. 6620

Practice Exercise 4.1

- | | | | |
|--------------------|----------------|----------------|----------------|
| 1. 25 % profit | 2. 200% profit | 3. 25 % profit | 4. 1 % loss |
| 5. 166.67 % profit | 6. \$5555.55 | 7. 4 eggs | 8. 968\$ |
| 9. 8% | 10. 22.22 % | 11. No gain | 12. 4.8 % gain |
| 13. 25% | 14. 23.5 % | 15. 22.22\$ | 16. 19.25% |

Practice Exercise 5.1

- | | | | | |
|------------|-----------|-------------|-------------|------------|
| 1. 8 : 5 | 2. 5 : 11 | 3. 31 : 12 | 4. 2 : 1 | 5. 80 : 49 |
| 6. 3/8 | 7. 3/7 | 8. 2340/690 | | |
| 9.i) 15/20 | ii) 30/36 | iii) 42/24 | iv) 225/100 | v) 42/12 |

Practice Exercise 5.2

- | | | | | |
|----------|------------|-------------|------------------|------------|
| 1. 3 : 7 | 2. 40 : 59 | 3. 17 : 25 | 4. 1 : 22 | 5. 24 : 29 |
| 6. 7/40 | 7. 6/15 | 8. 5 : 2.25 | 9. $\frac{3}{4}$ | 10. 11/2 |

Practice Exercise 5.3

- | | | | |
|---------------------------|-------------------------------------|--------------|-----------------------------|
| 1. 24 , 96 | 2. 20 , 80 | 3. 20 and 28 | 4. 900 , 1200 , 1500 , 1800 |
| 5. 3600 | 6. $1/\pi$ [$\pi = 22/7$ or 3.14] | 7. 43/20 | |
| 8. 2 \$1 , 3 \$10 , 8 \$5 | 9. 8 : 5 | | |
| 10. 35 , 77 | | | |

Practice Exercise 5.4

- | | | | |
|-------------------|-------------------|-------|--------|
| 1. $\frac{4}{3}$ | 2. $\frac{65}{8}$ | 3. 40 | 4. 224 |
| 5. $\frac{50}{7}$ | 6. $\frac{14}{3}$ | | |

Practice Exercise 5.5

- | | | | |
|---------------|--------------|--------------|------------|
| 1. 52.5\$ | 2. 20 apples | 3. 16 men | |
| 4.i) 79.2 deg | ii) 5 : 3 | iii) 5 : 9 | iv) 1000\$ |
| 5. 34 litres | 6. 5 : 9 | 7. 24 and 15 | |

Exercise 6.1

- | | | | | |
|------------|-------------|------------|------------|--------------|
| 1. 2 hours | 2. 80 miles | 3. 5 hours | 4. 4 hours | 5. 12.7 kmph |
|------------|-------------|------------|------------|--------------|

Practice Exercise 6.2

1. 15 2. 50 3. 125 4. 150 5. 64.8
6. 198 7. 216 8. 151.2

Practice Exercise 6.3

1. 240 2. 5 hours 3. 40 m 4. 140 m
5. 10 hours 6. 100 m 7. 10 km 8. When A has travelled 100 km
9. 20 km 10. 20 km 11. 20 km 12. 4 hours after B started
13. 10 km 14. After 15 hours 15. 240 km
16. 5 hours 17. Once 18. Once 19. 1 hour 40 min
20. 4 hours

Exercise 6.4

1. 28 km/hr 2. 102 km/hr 3. 4 hours 4. 9 steps

Exercise 7.1

1. $10/3$ days 2. 28 days 3. 50 % 4. 36 %
5. 11 days 6. 30 days 7. $100/9$ days
8. i) $400/13$ days ii) $400/9$ days 9. 150 days
10. 240 days

