

Entrance Essentials

Geometry workbook

Welcome to the world of geometry. Do you find geometry challenging? Do you find it difficult to visualize a cube, sphere, pyramid or any other 3d figure?

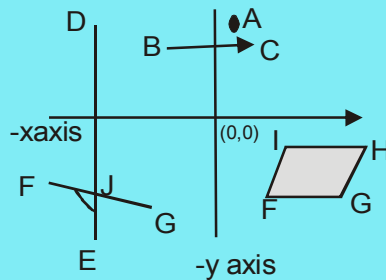
Chill, Geometry is not as tough as many proclaim it to be. All you need to do is spend time going through this workbook. Through this book you will understand the nuances of lines, triangles, circles, quadrilaterals... Feel free to email your queries to urmentor@semanticslearning.com for continuous support.

Enjoy learning

1. Lines and angles

Lines, angles, points, rays, planes are the fundamental concepts in geometry. Let's devote some time recognising these elements.

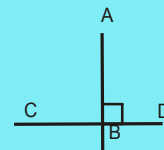
List the elements in the figure



- Line :
- Point :
- Ray :
- Angle :
- Plane :
- Origin :
- Axis :

Figural notations in geometry

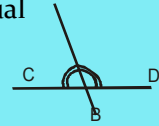
Perpendicular notation: A square box represents that one line is perpendicular to another.



Lines of equal dimension: Two lines of equal dimension can be represented by two or three lines cutting them.



Angles of equal dimension: Two or more angles of equal dimensions are represented by two or three curves.



1.1 Angles

Angles are classified into

acute (angles less than 90°)

obtuse (angles greater than 90°)

right (angles equal to 90°)

adjacent (angles next to each other)

supplementary (sum of angles = 180°)

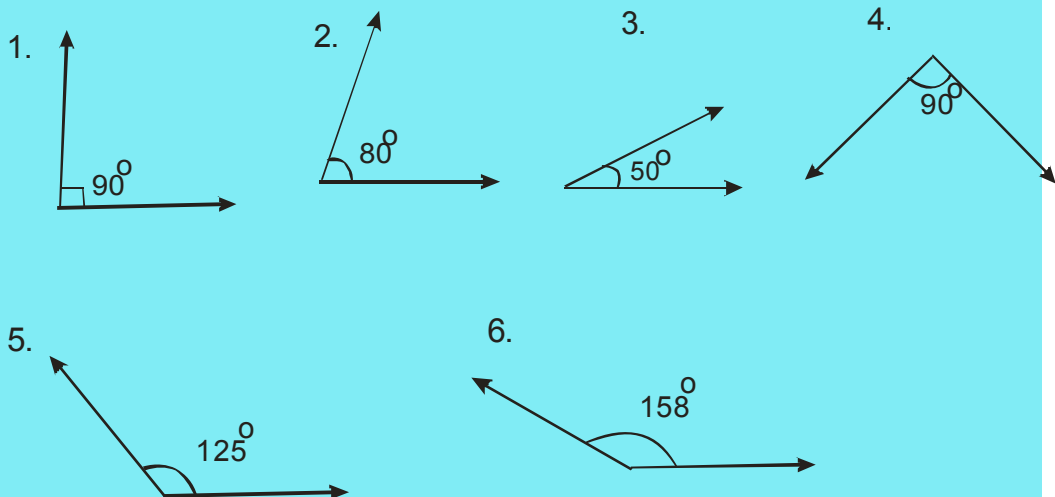
complementary (sum of angles = 90°)

vertically opposite angles (angles opposite to each other)

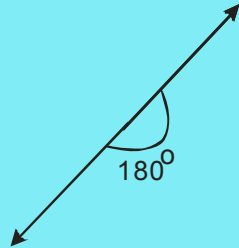
Interior angles (angles within a figure)

Exterior angles (angles outside the corresponding figure)

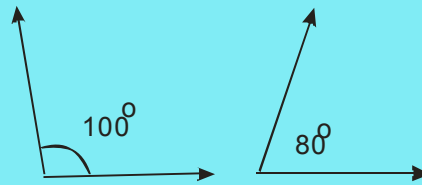
Identify the type of angle



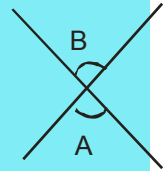
7.



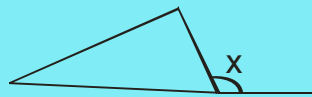
8.



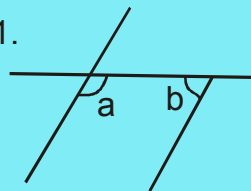
9.



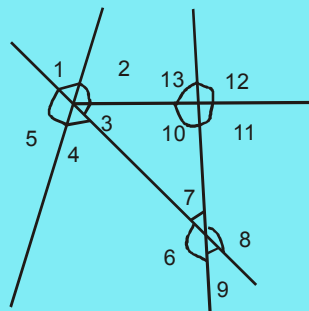
10.



11.



List the pair of vertically opposite angles which are equal to each other

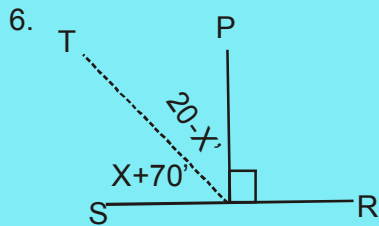
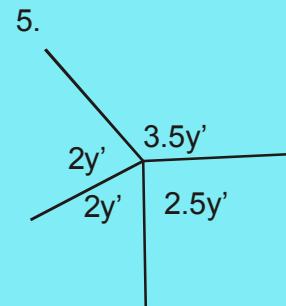
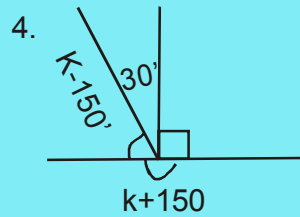
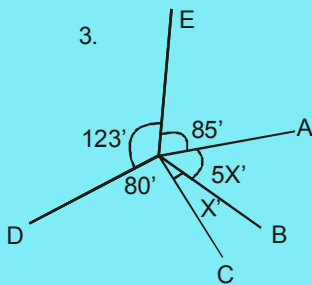
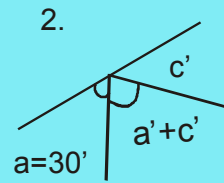
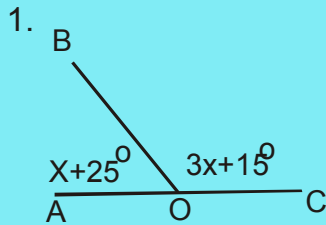


1.2 Find the unknown angles

The unknown angles can be calculated using the following facts

1. Sum of angles on a straight line = 180
2. Vertically opposite angles are equal
3. Sum of angles about a point = 360

Find the unknown angles using the above 3 rules



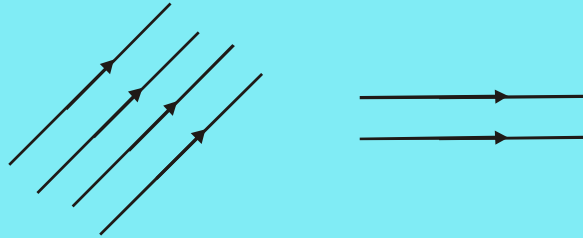
1.3 Parallel lines

Two lines are supposed to be parallel if

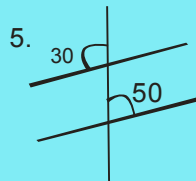
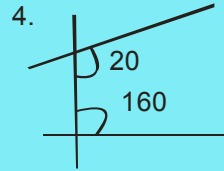
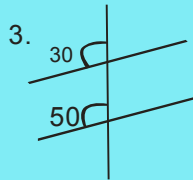
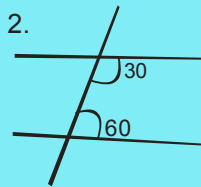
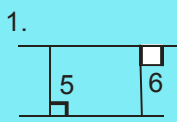
- The perpendicular distance between them is constant
- They never meet
- The sum of the interior opposite angles equal to 180

The geometrical notation representing parallel lines is two arrows on two lines. If more than two lines are parallel then all the parallel lines are

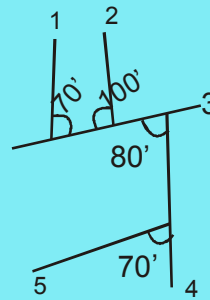
represented with arrows.



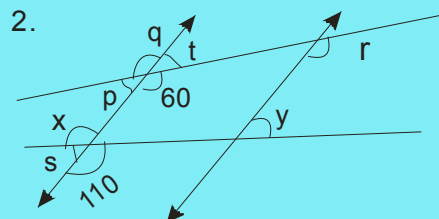
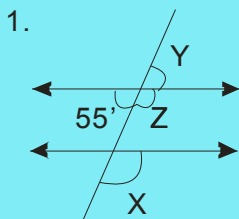
Determine whether these lines are parallel

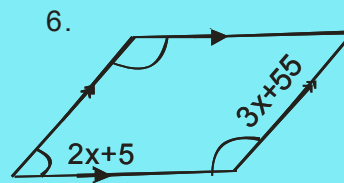
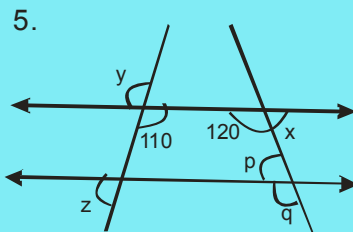
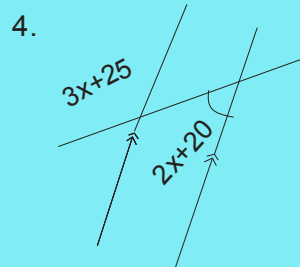
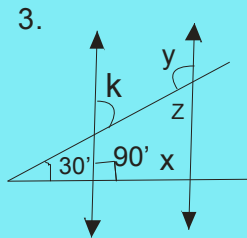


6. Which pair of lines is parallel?



7. Find the unknown parameter(s)





2. Triangles

A three sided figure made by joining three points is called a triangle.

The triangle comprises

Base(any side of the triangle)

Apex(any vertex of the triangle)

Altitude(height/ perpendicular line joining the apex and the opposite side)

Interior angles(angles present within the triangle)

Exterior angles(angles present outside the triangle)

Median(line which bisects the base)

Angle bisector(line which bisects the angle)

Circum-circle(circle passing the three vertices)

Circum-centre(centre of the circum-circle)

In-circle(circle which touches the three sides of the triangle internally)

In-centre(centre of the in-circle)

Identify the parts from the figure below

Apex :

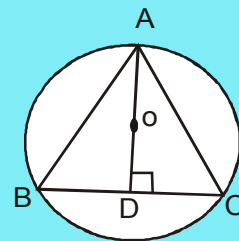
Altitude :

Sides :

Base :

Circum-centre :

Circum-circle :



Now identify the parts of the figure below

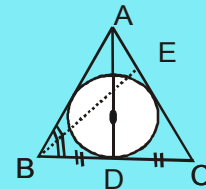
Median :

In-circle :

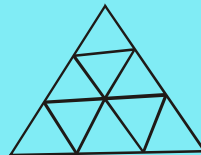
In-centre :

Angle bisector :

Two equal angles :



Count the number of triangles in the figure presented below



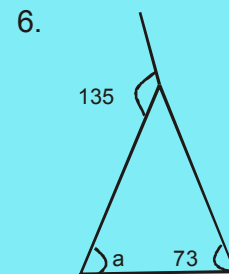
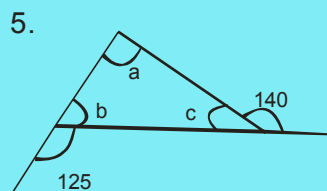
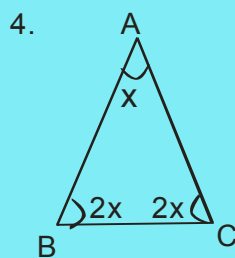
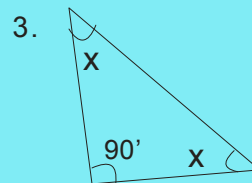
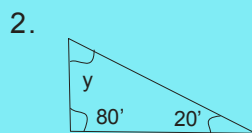
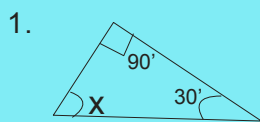
2.2 Angles and triangles

In a triangle ABC,

the sum of the interior angles is equal to 180.

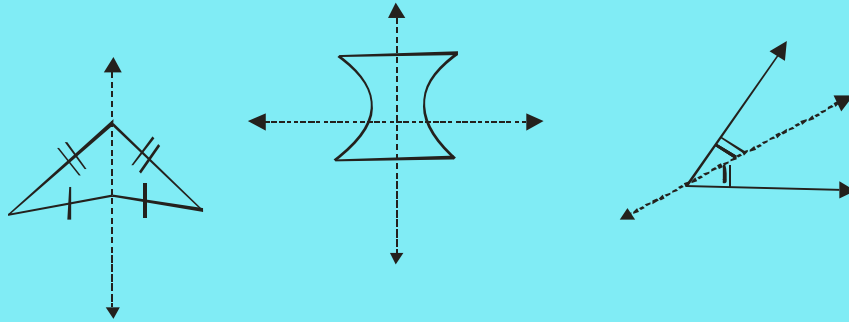
$$\angle ABC + \angle BAC + \angle ACB = 180^\circ$$

Solve for the unknown variable

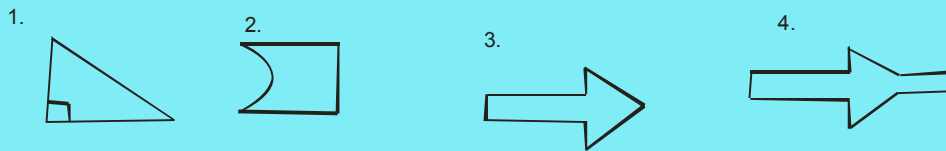


2.3 Symmetry of figures

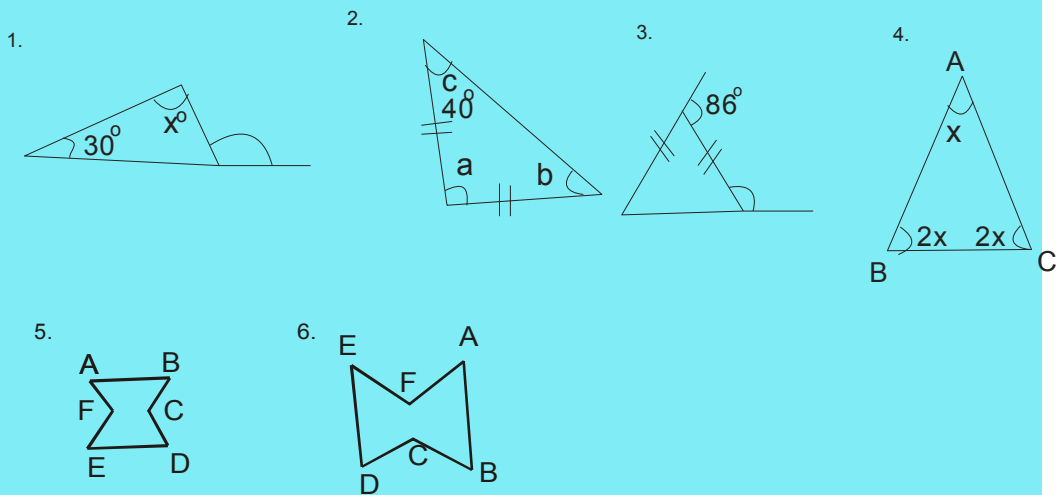
In geometry it is necessary to orient the figure about an axis, draw mirror images of figures, rotate the figure clockwise / anticlockwise.etc.. The following figures are symmetrical about their axes (represented in dashes)



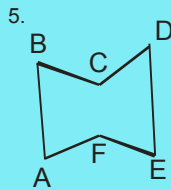
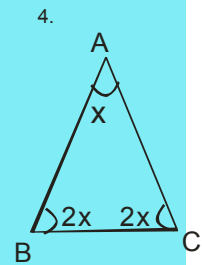
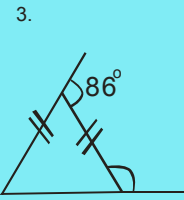
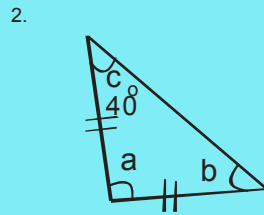
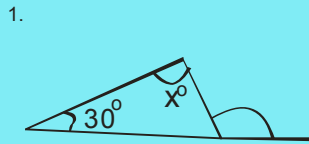
Draw the mirror image of the figures presented below



Rotate the figure clockwise by 90° and draw them



Rotate the figures anticlockwise by 90° and draw them



2.4 Congruence and symmetry

Triangles are said to be symmetric (the triangles are symmetric about an axis) if

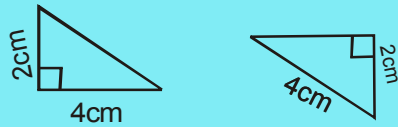
- Two angles are equal (AA)

Triangles are said to be congruent (the triangles are symmetric about an axis and their areas are equal) if

- Two angles and one side are equal (ASA)
- Two sides and one angle are equal (SAS)
- Three sides are equal (SSS)
- Three angles are equal (AAA)

State whether the following triangles are congruent or not

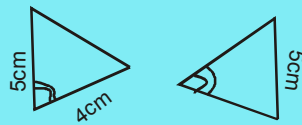
1.



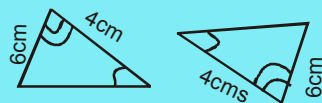
2.



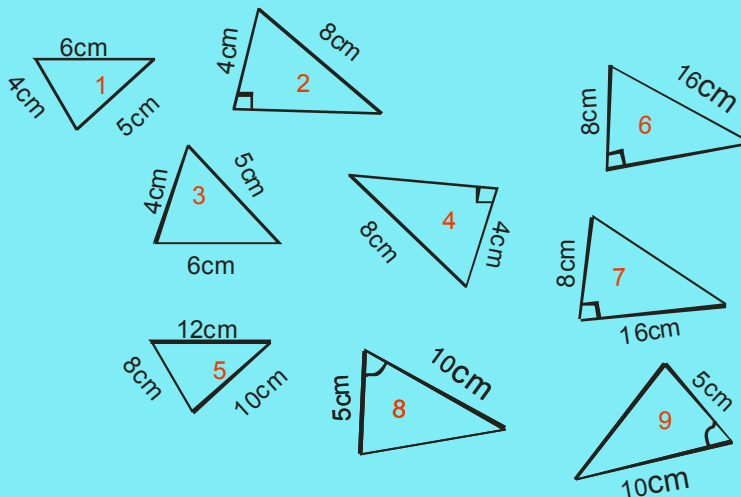
3.



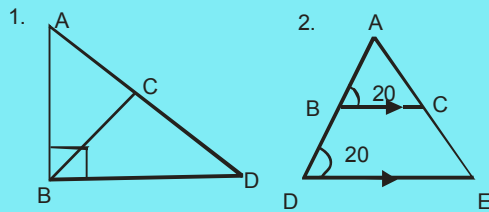
4.



Identify the congruent or similar pairs of triangles.



Identify the similar pairs of triangles in each of the figures



2.5 Pythagoras theorem

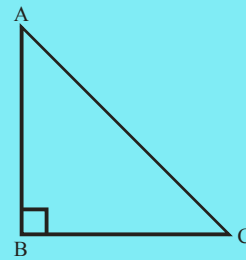
In a right angled triangle, the three sides are related by the Pythagoras theorem

AC is the Hypotenuse, BC and AB are side 1 and side 2 of the triangle then,

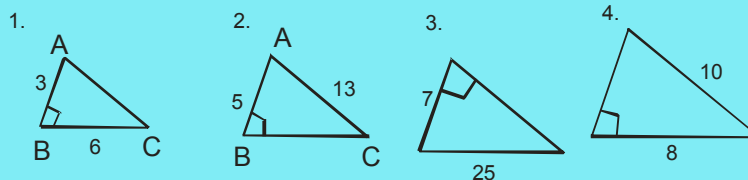
$$\text{Hypotenuse}^2 = \text{side } 1^2 + \text{side } 2^2$$

$$AC^2 = AB^2 + BC^2 \quad BC^2 = AC^2 - AB^2$$

$$AB^2 = AC^2 - BC^2$$



Find the third side of the triangle



2.6 Conversion of word data into a geometric diagram

In majority of the geometry based problems it is necessary to convert the word data into a geometric diagram.

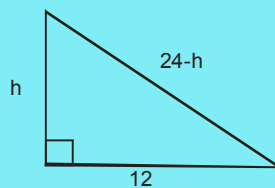
Remember

- To represent all the data into the figure
- To mark the sides/angles, which are equal, with necessary notations
- To represent parallel lines with necessary notations
- To represent the 90° angle whenever the situation demands
- Poles, trees, buildings are perpendicular to the ground

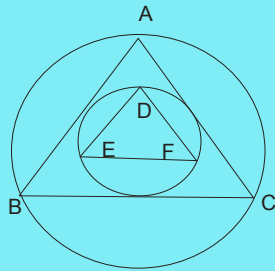
Diagram the following problems. Don't worry about the answers.

Sample problems:

1. A tree of height 24 m on a roadside broke at a certain height and fell in such a way that its top touched the other edge of the road. If the breadth of the road is 12m, then what is the height at which the tree broke?



2. Circle C_1 circumscribes an equilateral triangle ABC whose area is $16\sqrt{3}$ sq units. Circle C_2 inscribes the same triangle. Now circle C_2 circumscribes a smaller equilateral triangle DEF. What is the area of triangle DEF?



Now you try

3. If $A(2,3)$, $B(-2, -3)$ and $C(1,2)$ are three vertices of a triangle, which vertex comprises the largest angle?
4. A ladder 65' long is leaning against a straight wall. Its lower end is 25' from the bottom of the wall. How much farther away will it be if the upper end is moved down by 8'?
5. I walk 4 km to the North, then 3 km to the East. How far am I from the starting point?
6. A pole 25 dm high is tied to a peg, 45 dm from foot of pole by a rope .Find length of rope?
7. Looking East I see a church which is 2 km far. Looking North I see a tower 1.5 km away. How far the tower and the church are apart?
8. In a river 48 m wide, a man was carried 16 m downstream. Find the distance between the starting point and the landing point?

9. In $\triangle ABC$, BO & CO bisect the $\angle B$ & $\angle C$. Find $\angle BOC$ in term of angle A .
10. A gardener is placing an exotic flower arrangement. He wants to plant rose flower bed at a point equidistant from the vertices of his lawn which is in the shape of an equilateral triangle of side 10 cms. What is length of the rose flower bed?
11. AD is the medium of triangle ABC . A square is constructed with AD as the side. Find the area of the square given $AB = 5$ $AC = 6$ and $BC = 4$ cms.
12. A 20-foot ladder leaning against a vertical wall with the base of the ladder 10 feet away from the wall is pulled 2 feet further out from the wall, causing the top of the ladder to drop x feet. What is the value of x ?
13. The foot of a ladder leaning against a vertical wall is 5 feet away from the wall. If the top of the ladder is moved 1 feet vertically up along the wall, the ladder becomes vertical. What is the length of the ladder?
14. BC is trisected at D , and DE and DF are drawn parallel to AB and AC respectively. What is the ratio of the area of the parallelogram $AFDE$ to that of the triangle ABC ?

15. Mr. and Mrs. Shaw travel from city A to city B . They take a break City C in between city A and city B. Somewhere in-between City A and City C, Mrs. Shaw asks, “How far have we traveled?” , Mr. Shaw replies “Half as far as the distance from here to city C”. Somewhere between city C and city B, exactly 200 kms from the point where she asked the first question, Mrs. Shah asks “How far do we have to go?” Mr. Shaw replies “half as far as the distance from city C to here.” What is the distance between cities A and B is, in kms?

3. Circles

A circle is the only geometrical figure without a side.

The circle comprises

Centre(a point which is equidistant to any point on the boundary)

Radius(the distance between centre and the boundary)

Circumference(The line forming the outer boundary of the circle)

Diameter(The line passing through the centre and touching two points on the circumference)

Tangent(line touching the circumference, outside the circle, at a single point)

Chord(line joining any two points on the circumference within a circle, the longest chord is the diameter)

Secant(A chord which is extended to a point outside the circle)

Major segment(When the chord splits the circle into two segments, the greater region is the major segment)

Minor segment(When the chord splits the circle into two segments, the smaller region is the minor segment)

Angle subtended by chord(the apex angle of a triangle formed by joining the chord and any point on the circumference)

Arc(A portion of the circumference)

Sector(A portion of the circle which comprises two radii and an arc)

Centre angle(angle subtended by the chord at the centre of the circle)

Identify the various portions of the figure

Chord :

Tangent :

Secant :

Sector :

Angle subtended:

by the chord

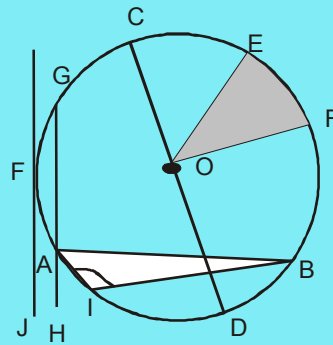
Central angle :

Arc :

Major segment :

Minor segment :

Radius :



Centre :

Diameter :

3.1 Area and circumference of a circle

The area of a circle is given by πr^2 where r is the radius of the circle

The circumference of a circle is given by $2\pi r$ where r is the radius of the circle.

Find the area and the circumference of a circle whose

1. radius is 4cms
2. radius is 8 cms
3. diameter is 5 cms

3.2 Conversion of word data into geometrical data involving circles

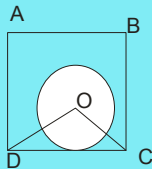
In majority of the geometry based problems it is necessary to convert the word data into a geometrical figure.

Remember

- To mark the centre of the circle
- To distinguish the major segment from the minor segment while drawing a chord
- To represent the problem in terms of radius or the diameter

Diagram the following problems. Don't worry about arriving at an answer in this exercise.

1. ABCD is a square. CD is tangential to the circle with centre O. O is at a point such that $OC = ED$. Calculate the radius of the circle, given that the area of circle is $\frac{\pi}{3}$ times the area of square.



Now try diagramming these problems

2. What is the maximum area(in cm^2) that can be enclosed within a string of length 10 cms?
3. There is a circular pizza with negligible thickness that is cut into 'x' pieces by 4 straight line cuts. What is the maximum and minimum value of 'x' respectively?
4. Two straight roads intersect at an angle of 90° . A bus on one road is 2 kms away from the intersection while a car on the road is 3 kms away from the intersection. Then what is the shortest distance between the two vehicles?
5. A circle is inside an equilateral triangle of side 6. Then what is the area of

any square inscribed in the circle?

6. If four circles of radius 1 unit each are arranged such that a square is formed when their centers are joined, what is the area of the region lying trapped between the circles, but not part of any circle?
7. If the circumference in cm of a circle is equal to its area in cm^2 , find the area of the largest triangle that can be inscribed in the circle.
8. Two circular cylinders are connected tightly with a belt. The distance between their centers is 10m and their radii are 3 metres and 4 metres respectively. The belt covers exactly half the circumference of both the cylinder. What is the length of the belt?
9. XYCB is a cyclic quadrilateral (A quadrilateral with all its 4 vertices on the circumference of a circle). XB and YC are produced to meet at P. XY and BC are produced to meet at Q. If $\angle XPC = 40^\circ$, $\angle BQX = 30^\circ$ what is the value of $\angle BXY$ in degree?
10. Two identical circles intersect so that their centers, and the points at which they intersect, form a square of side 1 cm. What is the area in sq.cm of the portion that is common to the two circles ?

4. Polygon

Any n sided figure is called a polygon

A 3 sided figure is called a triangle.

A 4 sided figure is a quadrilateral.

The different types of quadrilaterals are

Parallelogram (Opposite sides are equal and parallel)

Trapezium (two sides are parallel but not of the same length, the other two sides are not related to each other)

Rectangle (opposite sides are equal and parallel and angle between them is 90°)

Rhombus (all 4 sides are equal and parallel but angle between them is not 90°)

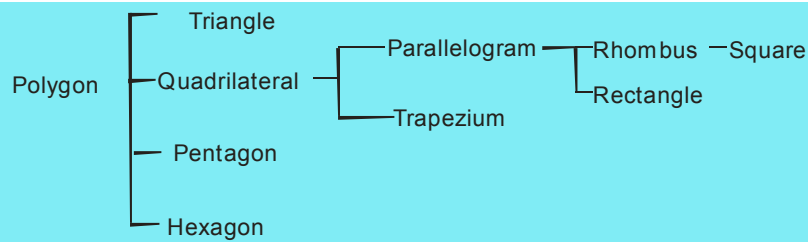
Square (all 4 sides are equal and parallel and angle between them is 90°)

A 5-sided figure is called a pentagon.

A 6-sided figure is called a hexagon.

There are also 7 sided, 8 sided, 9 sided, 10 sided figures....

The hierarchy is as follows



All squares are rhombuses but not all rhombuses are squares

All rectangles are parallelograms

A regular polygon has all its sides equal

A square is a regular quadrilateral

4.1 Conversion of word based problems involving polygons into geometric figures

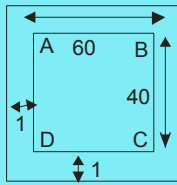
In majority of the geometry based problems it is necessary to convert the word data into a geometrical diagram.

Remember

- To distinguish between a square, parallelogram and rectangle
- To mark the dimensions of the polygon
- To mark the sides/angles which are equal to each other
- To distinguish between the height of the quadrilateral and the side of the quadrilateral

Diagram these problems. Don't worry about solving the problems.

1. A lawn is in the shape of a rectangle of length 60 m and width 40 m. Outside the lawn there is a footpath of uniform width 1m bordering the lawn. Then what is the area of the path



2. A square is drawn joining the midpoint of the sides of a given square. A third square is drawn inside the second square in the same way and the process continues indefinitely. If each side of the first square is 4 cms, then what is the sum of the area of all the squares?
3. A rectangular box is 10 inches wide, 10 inches long and 5 inches high. What is the greatest possible distance in inches between any two points in the box?
4. A conical flask of diameter 10 inches and slant height of 13 inches is filled with water. Half the contents of the flask are poured into a cylindrical measuring jar of radius 5 inches. The height of water (inches) in the cylinder is
5. Two poles of height 6m and 11m stand vertically on a plane ground. If the

distance between their feet is 12m, then what is the distance between their tops?

6. How many balls can be placed around a ball so that the latter touches each of the former balls given that all the ball are identical?
7. An ice cream cone is the union of a right circular cone and a hemisphere that has the same circular base as the cone. Find the volume of ice cream if the height of the cone is 9 cm and the radius of its base is 2.5 cms.
8. A four sided plot of land has two mutually perpendicular sides of 48m and 64m. The other two sides are equal and each is of length 50m. What is the area of the plot if one of the angles of the quadrilateral is greater than 180° ?
9. ABCD is a parallelogram. G is the mid-point of AD and $DE = EF = FC$. What is the ratio of the area of $\triangle AGB$ to that of $\triangle FBC$?
10. A rectangular plot of land is surrounded by a footpath of uniform width of 5m. If the area of the footpath is 800 sq m, then what is the perimeter of the plot?
11. ABCD is a square and PQRS is a rectangle. S is the midpoint of BC while P divides the side DC in the ratio 1:3. What is the ratio of the areas of quadrilaterals ABCD and PQRS?

12. Four points A, B, C and D lie on a straight line in the X-Y plane, such that $AB=BC=CD$, and the length of AB is 10 meter. An ant at A wants to reach a sugar particle at D. But there are insect repellents kept at points B and C, the ant would not go within one meter of any insect repellent. The minimum distance in meters the ant must traverse to reach the sugar particle is?
13. Rectangular tiles each of size 70 cm by 30 cm must be laid horizontally on a rectangular floor of size 1100 cm by 1300 cm, such that the tiles do not overlap. A tile can be placed in any orientation so long as its edges are parallel to the edges of the floor. No tile should overshoot any edge of the floor. What is the maximum number of tiles that can be accommodated on the floor?

Answers

Exercise 1.1

Line : FG
Points : A
Rays : BC
Angles : FJE, GJE
Origin : (0,0)
Axis : X axis, Y axis

1. Right angle
2. Acute angle
3. Acute angle
4. Right angle

5. Obtuse angle
6. Obtuse angle
7. Straight angle
8. Supplementary angles.
9. Vertically opposite angle
10. Exterior angles
11. Interior angles

Vertically opposite angles.

$$\angle 1 = \angle 4$$

$$\angle 2 = \angle 5$$

$$\angle 6 = \angle 8$$

$$\angle 7 = \angle 9$$

$$\angle 11 = \angle 13$$

$$\angle 12 = \angle 10$$

Exercise 1.2

1. $x = 35^\circ$

2. $c = 60^\circ$

3. $x = 12$

4. $k = 30^\circ$

5. $y = 36^\circ$

6. $x = 0$

Exercise 1.3

1. Yes
2. Yes
3. No
4. No
5. No

6. 1 and 2 are parallel

7. (1) $x = 55^\circ$, $y = 55^\circ$, $z = 125^\circ$ (2) $q = 60^\circ$, $p = 120^\circ$, $r = 60^\circ$, $s = 70^\circ$, $x = 110^\circ$, $y = 70^\circ$

(3) $x = 90^\circ$, $y = 120^\circ$, $z = 60^\circ$, $k = 60^\circ$ (4) $x = -5$ (5) $x = 60^\circ$, $y = 110^\circ$, $z = 70^\circ$ P = 60° , $q = 120^\circ$ (6) $x = 24^\circ$

Exercise 2.1

Apex - B

Attitude - AD

Sides - AB, BC, CA

Base - BC

Circumcentre - o

Median - AD

Incircle - Circle inside

Incentre - o

Angle bisector - AD

Equal angle - $\angle ABD$ and $\angle ADC$

Number of triangle : 13

Exercise 2.2

1) $x = 60^\circ$

2) $y = 80^\circ$

3) $x = 45^\circ$

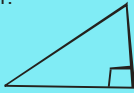
4) $x = 36^\circ$

5) $a = 85^\circ, b = 55^\circ, c = 40^\circ$

6) 62°

Practice Exercise 2.3

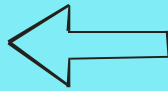
1.



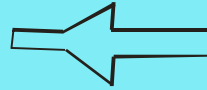
2.



3.



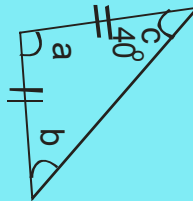
4.



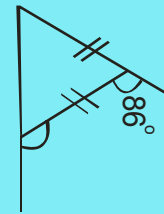
1.



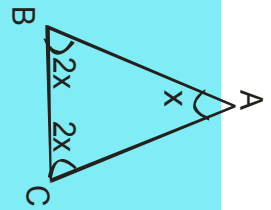
2.



3.



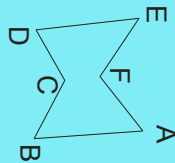
4.



5.



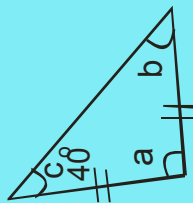
6.



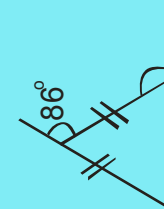
1.



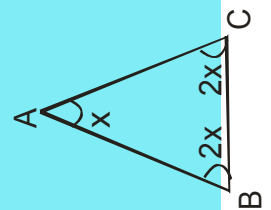
2.



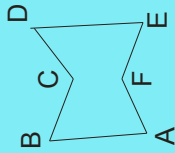
3.



4.



5.



Exercise 2.4

1. Yes

2. Yes

3. No

4. Yes

1 and 3 are congruent

2 and 4 are congruent

8 and 9 are congruent

2 and 6 are similar

4 and 6 are similar

Similar triangles

1) $\triangle ABC$ & $\triangle BCD$

2) $\triangle ABC$ & $\triangle ADE$

Exercise 2.5

1. $\sqrt{45}$

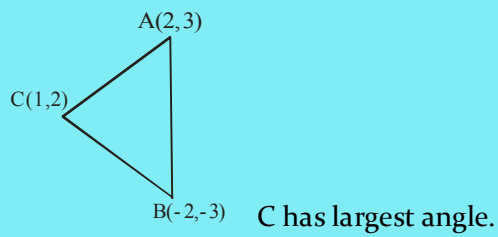
2. 12

3. $\sqrt{24}$

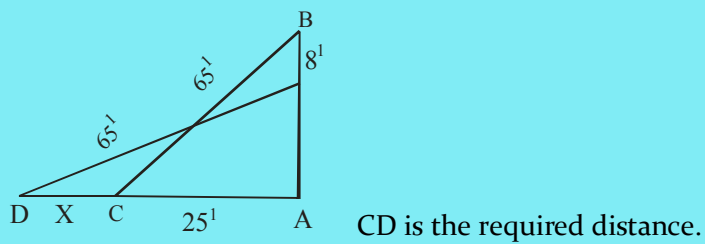
4. 6

Exercise 2.6

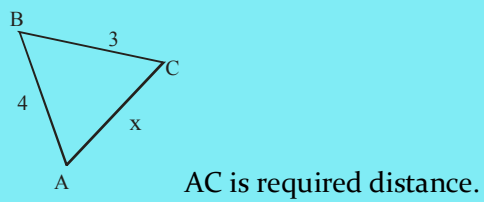
3)



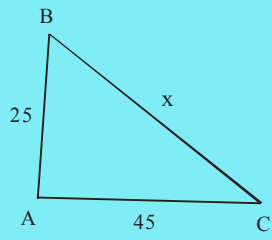
4)



5)

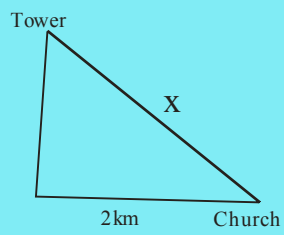


6)

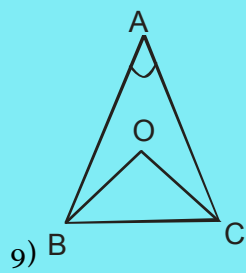
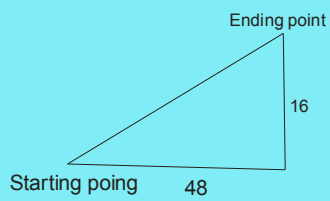


x is length of rope.

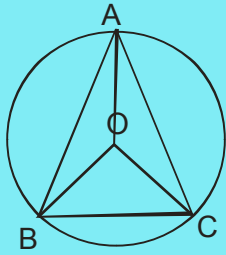
7)



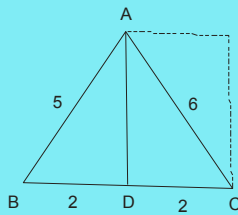
8)



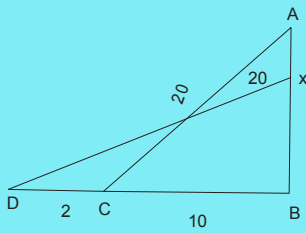
10)



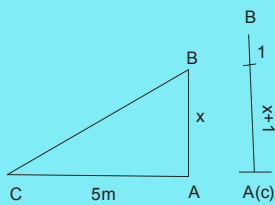
11)



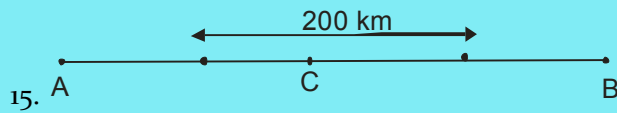
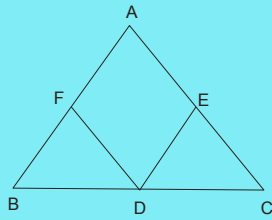
12)



13)



14)



Exercise 3.1

Chord : AB, BH, CD, GA.

Tangent : FT

Secant : GH

Sector : OEF

Angle subtended: $\angle AIB$

by the chord

Central angle : $\angle EOF$

Arc : Arc EF

Major segment : ICB

Minor segment : IDB

Radius : OD, OC

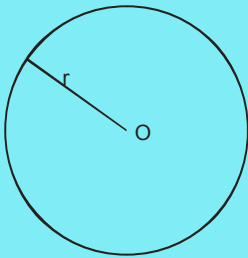
Centre : O

Diameter : DC

Area and circumference.

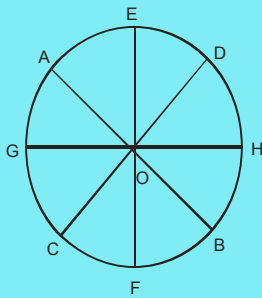
1. Area = 50.28 sq.cm, circumference = 25.14
2. Area = 201.14 sq.cm., circumference = 50.28
3. Area = 19.64 sq.cm, circumference = 15.714

Exercise 3.2

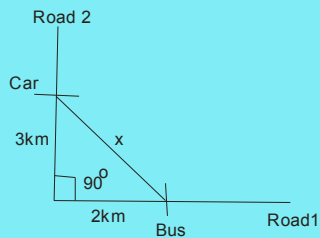


2.

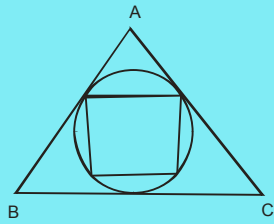
3.



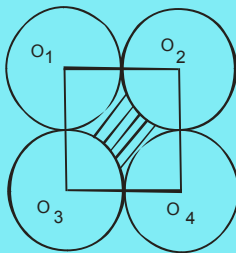
4.



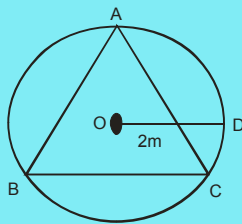
5.



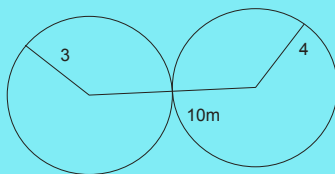
6.



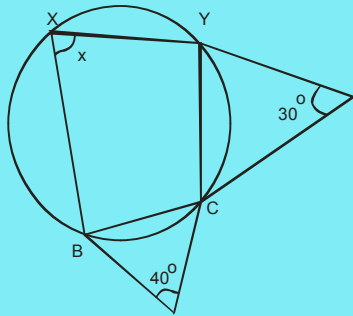
7.



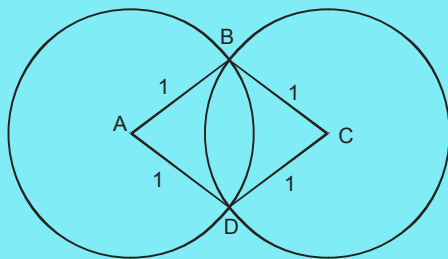
8.



9.

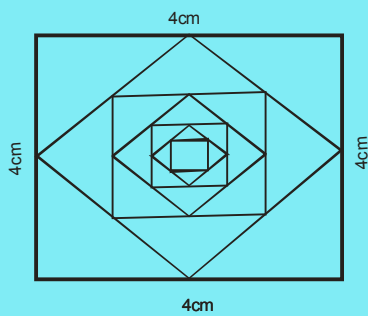


10.

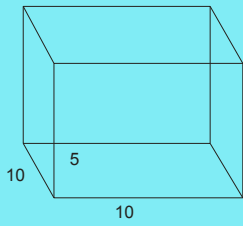


Exercise 4.1

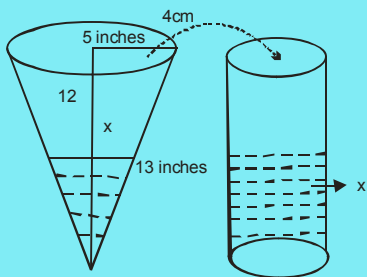
2.



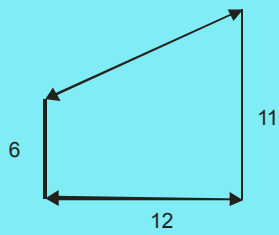
3.



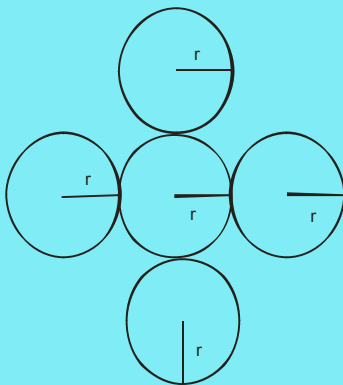
4.



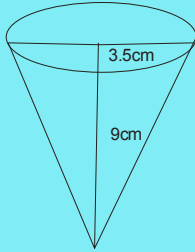
5.



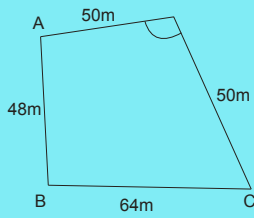
6.



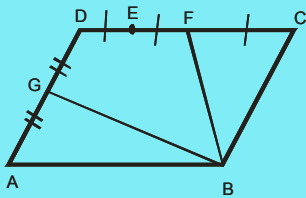
7.



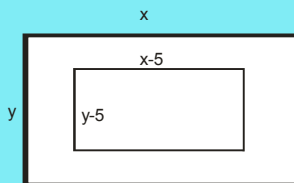
8.



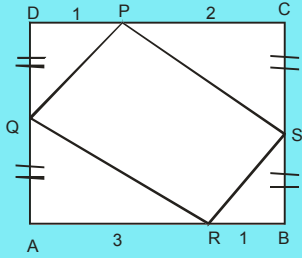
9.



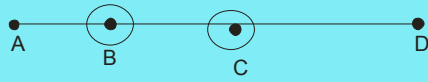
10.



11)



12.



13.

